ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

3. Proposed by Professor H. A. WOOD, A. M., Hoboken, New Jersey.

If $x^6-y^6=665$, and $x^3y+xy^3=78$, find x and y.

Solution by Professor G. B. M. ZERR, A. M., Principal of Schools, Staunton, Virginia.

Let $x^2-y^2=s$, $xy=p$. Then $x^6-y^6=s^3+3p^2s-665$, $\therefore p^2=\dfrac{665-s^3}{2s}$....(1). Also, $p\sqrt{s^2+4p^2}=78$, or $p^2s^2+4p^4=6084$....(2).

(1) in (2) gives, $s^6-3325\,s^3-54756s^2+1768900=0$.

$\therefore$ $(s-5)(s^5+5s^4+25s^3-3200s^3-70756s^3-353780)=0$.

$\therefore$ $s=5$. $\therefore$ $x^2-y^2=5$, $xy=6$, and now $x=\pm3$, $y=\pm2$.

Solved also by *P. S. Berg*, and Professors *Scheffer* and *Whitaker*.

4. Proposed by L. E. PRATT, Tecumseh, Nebraska.

If Σm, Σm^3, Σm^5,....Σm^{2n-1} are the sums of the 1st, 3rd, 5th,..... $(2n=1)$th powers of the first m natural numbers, prove that $n\Sigma m^{2n-1}+\dfrac{n(n-1)(n-2)}{\underline{|3}}\Sigma m^{2n-3}+\dfrac{n(n-1)(n-2)(n-3)(n-4)}{\underline{|5}}\Sigma_m{}^{2n-5}+\ldots\ldots=2^{n-1}\Sigma^n_m$.

Solution by the Proposer.

Assume the natural series 1, 2, 3, 4, 5,....$(m-1)^n$,....m^n,....$(m+1)^n$ ....(1). That portion of (1) included between $(m-1)^n$ and $(m+1)^n$ may be written

$$\frac{\{(m^n-1)-(m-1)\}\{(m-1)^n+m^n\}}{2},\ m_n,\ \frac{\{(m+1)^n-(m^n+1)\}\{m^n+(m+1)^n\}}{2}\ \ldots.(2).$$

Collecting all the terms having the form m^n contained in (2), we have $\dfrac{m^n\{(m+1)^n-(m-1)^n\}}{2}$....(3).

This expression is the general term of a series, involving the first m terms of (1), whose sum, as may be seen by changing m into $m-1$, $m-2$,...., is $\dfrac{m^n(m+1)^n}{2}$, or $2^{n-1}\Sigma^n_m$. If we now expand (3), we have

$$n\,m^{2n-1}+\frac{n(n-1)(n-2)}{\underline{|3}}m^{2n-3}+\frac{n(n-1)(n-2)(n-3)(n-4)}{\underline{|5}}m^{2n-5}+\ldots.(4).$$

Each term of (4) may be regarded as a type-term of a series, where m may have the same values as in (3).

$$\therefore\ n\,\Sigma m^{2n-1}+\frac{n(n-1)(n-2)}{\underline{|3}}\Sigma m^{2n-3}+\ldots.=2^{n-1}\Sigma^n_m$$

[By Σ^n_m is meant $(\Sigma_m)^n$. The latter is the preferable way of writing it. Several contributors misunderstood Σ^n_m as given by the proposer.—Editor.]

5. Proposed by WILLIAM E. MAY, 500 Union St., Knoxville, Tennessee.

A, B, C went to market, each with 10,30, and 50 eggs, respectively. On their

way to market, they agreed to sell their eggs at the same price(s) per dozen so as to realize an equal integral number of cents. How much did they receive?

I. Solution by A. L. FOOTE, No. 80, Broad St., New York City.

Let A sell 1 egg at 2 cents, and the remaining 9 at x cents, realizing $(2+9x)$ cents; let B sell $(1+n)$ eggs at 2 cents and $30-(1+n)$, or $29-n$ at x cents, realizing from the two sales $(2+2n+29x-nx)$. But by the conditions we have, $(2+9x)=(2+2n+29x-nx)$, or by solving, $x=\frac{2n}{n-20}$; the least value of n to render n integral is 21, consequently $x=42$, and we have, the sales of A and B as follows:

A 1 at 2cts. and 9 at 42cts.=380cts., B 22 at 2cts. and 8 at 42cts.=380cts. Now let C sell m eggs at 2cts. and $(50-m)$ at 42cts., and we have $2m+2100-42m=380$, or $40m=2100-380$, from which $m=43$ eggs at 2cts. and 7 at 42cts., making 380cts., as before. So the sales of each were,

A 1 egg at 2cts. and 9 at 42cts.=380cts.,
B 22 eggs at 2cts. and 8 at 42cts.=380cts.,
C 43 eggs at 2cts. and 7 at 42cts-=380cts.

N. B.—A variety of answers may be obtained by varying the suppositions.

[This egg puzzle problem is somewhat indefinite as published. The proposer afterwards changed the second part of the problem to read: "they agree to sell their eggs at the same rates per dozen, so as to obtain no fractional part of a cent from a sale and receive an equal integral number of cents." He assumes that at first they find the market slow for eggs, and were offered only $\frac{1}{7}$ of a cent per egg, or $\frac{12}{7}$cts. per dozen. At this price A sells 7, B 28, and C 49. Eggs then take a rise and are worth 3cts. per egg, or 36cts. per dozen, at which price they sell the remainder of their eggs. The sales of A, B, and C then stood as follows:

A 7 eggs at $\frac{1}{7}$cts. (1st sale)+3 at 3cts. (2nd sale)=10cts.,
B 28 aggs at $\frac{1}{7}$cts. (1st sale)+2 at 3cts. (2nd sale)=10cts.,
C 49 eggs at $\frac{1}{7}$cts. (1st sale)+1 at 3cts. (2nd sale)=10cts.

Prof. Whitaker remarks that the price may change nine times, but assuming that it changed once gives the condition that twice the number B sold at the first price equals the number A sold + the number C sold both at that price, a condition expressed by $2y=x+z$.—Editor.]

6. Proposed by L. E. PRATT, Tecumseh, Nebraska.

A vessel is to be filled with water by two pipes. The first pipe is kept open during m-nth of the time which the second would take to fill the vessel; then the first pipe is closed and the second is opened. If the two pipes had kept open together, the vessel would have been filled t hours sooner, and the first pipe would have brought in p-qth of the quantity of water which the second pipe really brought in. How long would it take each pipe alone to fill the vessel?

Solution by J. F. W. SHEFFER, A. M., Hagerstown, Maryland.

Let x and y designate the times in which the vessel would be filled by the two pipes, respectively, were each open by itself. The time during which the first pipe is kept open is $=\frac{m}{n}y$. $\therefore$ $1-\frac{my}{nx}$ is the part of the vessel to be